

WIGNER FUNCTION AND DECOHERENCE¹**Claus Kiefer**Fakultät für Physik
Universität Freiburg
Hermann-Herder-Straße 3
D-79104 Freiburg, Germany**Abstract**

I briefly review the role of the Wigner function in the study of the quantum-to-classical transition through interaction with the environment (decoherence).

My contribution to the Fifth Wigner Symposium in Vienna touches upon two of Wigner's fields of interest: His work on the interpretation of quantum theory [1] and his discussion of quasi-probability functions as encoded in the famous Wigner function [2]. As far as the interpretation is concerned, a crucial role in [1] is attributed to a result by Zeh [3] showing that it is usually very unrealistic to consider systems as being isolated from their natural environment. Except for small objects such as atoms and small molecules, they are quantum-entangled with a huge number of other degrees of freedom. This fact *must* therefore be taken into account to gain a proper interpretation of quantum theory.

Under ordinary circumstances, these correlations with the environment lead to the local appearance of classical properties – a process known as decoherence (see the comprehensive review [4], where all necessary details can be found). Amongst the most important features of decoherence is its ubiquitous, unavoidable, occurrence as well as its irreversible nature. Technically, decoherence is investigated through the local, reduced, density matrix after the degrees of freedom of the environment are traced out. It is important to emphasise that the interaction with the environment leads to the emergence of a distinguished local “pointer basis” which is stable in time and which *defines* the local property that is called “classical”. The process of decoherence has been experimentally monitored in quantum optics experiments [5].

¹To appear in the Proceedings of the Fifth Wigner Symposium, edited by P. Kasperkovitz (World Scientific, Singapore, 1998).

While a very weak coupling to the environment is usually sufficient to ensure classical behaviour, the local classical equations of motion only hold approximately if this coupling is not too strong. Otherwise, a large back reaction may result, which can disturb the local dynamics or even freeze it (Zeno effect). It is the discussion of this aspect for which the Wigner function is a useful tool (see Sect. 3.2.3 of [4]). It is used, in particular, to investigate the *correlations* which exist between position and momentum after the interaction with the environment is taken into account.

Some of the relevant examples are the following:

- *Localisation of objects.* This is an important example because it shows why macroscopic objects appear localised within quantum theory. The interaction with the environment can prevent the spreading of the wave packet even in cases where the mass is so small that it would otherwise be noticeable (such as for a small dust grain).
- *Decoherence of fields.* Macroscopic field strengths can become classical through interaction with charged fields. On the other hand, charges become classical quantities through interaction with electromagnetic fields.
- *Emergence of classical spacetime.* A quantised gravitational field would exhibit genuine quantum features. Due to its universal coupling with all other fields in Nature, the entanglement becomes so strong that at least the global gravitational degrees of freedom (such as the radius of the Universe) become classical to a high degree of accuracy. The notion of an (approximate) classical time parameter emerges through symmetry breaking [4].
- *Emergence of classical fluctuations in the early Universe.* This is of utmost importance for the structure formation. Quantum fluctuations arise in a natural way in the context of inflationary universe models, but only if they become classical can they be interpreted as providing the seeds for galaxy formation [6, 7].

In the above examples, one often encounters a wave function of the form

$$\psi(x, y) \propto \exp(iS(x)) \exp\left(-\frac{\Omega(x)}{2}y^2\right),$$

where Ω is complex, and x (y) is an abbreviation for the degrees of freedom of the system (environment). For example, in the case of QED, y may be the charged field degrees of freedom, while x may be the electric field. In many of the applications, ψ represents a squeezed state.

Integrating out the “irrelevant” degrees of freedom y , one gets the reduced density matrix, ρ , for the “relevant” degrees x . From ρ one can then construct the corresponding Wigner function in the standard way [2, 4]. In the Gaussian approximation, the width of ρ gives a quantitative measure for the remaining

coherence, while the corresponding width in the Wigner function is a measure for the correlation between position and momenta. Since one width is the inverse of the other, an “uncertainty relation” between coherence and correlation holds.

For the above state, the correlation between position and momentum is of the form $p_x = dS/dx + \dots$, where \dots denotes the back reaction of the y -degrees of freedom on the system x . In the QED case one finds, for example, that a constant electric field will be first slightly increased, but becomes weaker after particle creation comes into play. Analogous results hold for the above cited examples of the gravitational field and the fluctuations in the early Universe.

The topic “Wigner function and decoherence” is thus of great relevance in many branches of physics, covering such different fields like particle physics, quantum optics, and cosmology.

References

- [1] E.P. Wigner, *Philosophical Reflections and Syntheses* (Springer, Berlin, 1997).
- [2] E.P. Wigner, *Phys. Rev.* **40**, 749 (1932).
- [3] H.D. Zeh, *Found. Phys.* **1**, 69 (1970).
- [4] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, and H.D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 1996).
- [5] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J.M. Raimond, and S. Haroche, *Phys. Rev. Lett.* **77**, 4887 (1996).
- [6] D. Polarski and A.A. Starobinsky, *Class. Quantum Grav.* **13**, 377 (1996).
- [7] C. Kiefer and D. Polarski, in preparation.